Teoría de la Empresa

A partir de los materiales de Ianina y Máximo Rossi

Función de producción

$$q = f(X_{1,} X_{2,} \dots, X_n)$$

$$q = f(X_{1,} X_2)$$

Producto Medio

$$PMe_1 = \frac{f(X_{1,} X_2)}{X_1}$$

$$PMe_2 = \frac{f(X_{1,} X_2)}{X_2}$$

Producto Marginal

$$PMg_1 = \frac{\delta f(X_{1,} X_2)}{\delta X_1}$$

$$PMg_2 = \frac{\delta f(X_{1,} X_2)}{\delta X_2}$$

¿Cómo se relacionan el PMe y el PMg?

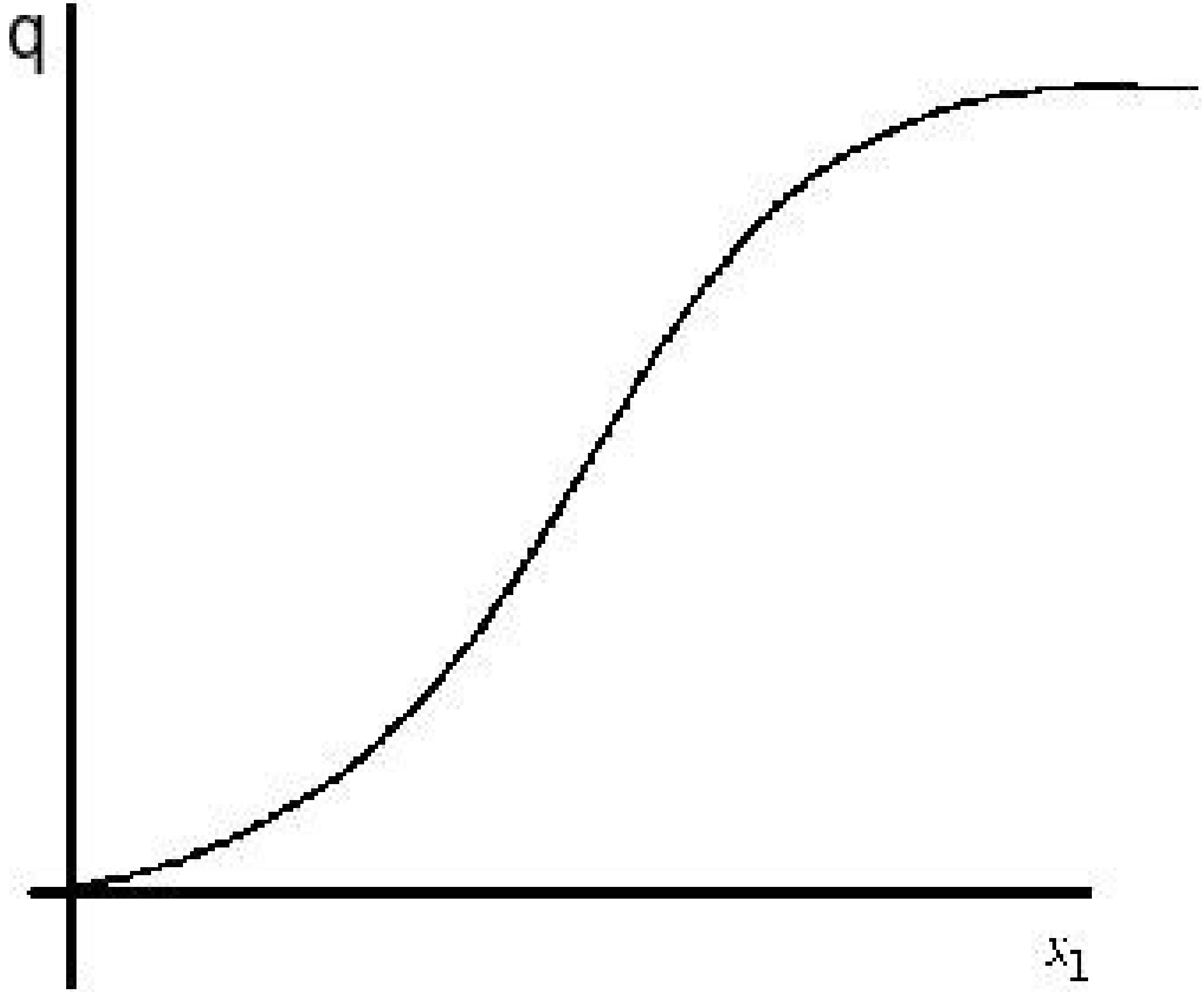
Vamos a buscar el nivel de producción que maximiza el producto medio

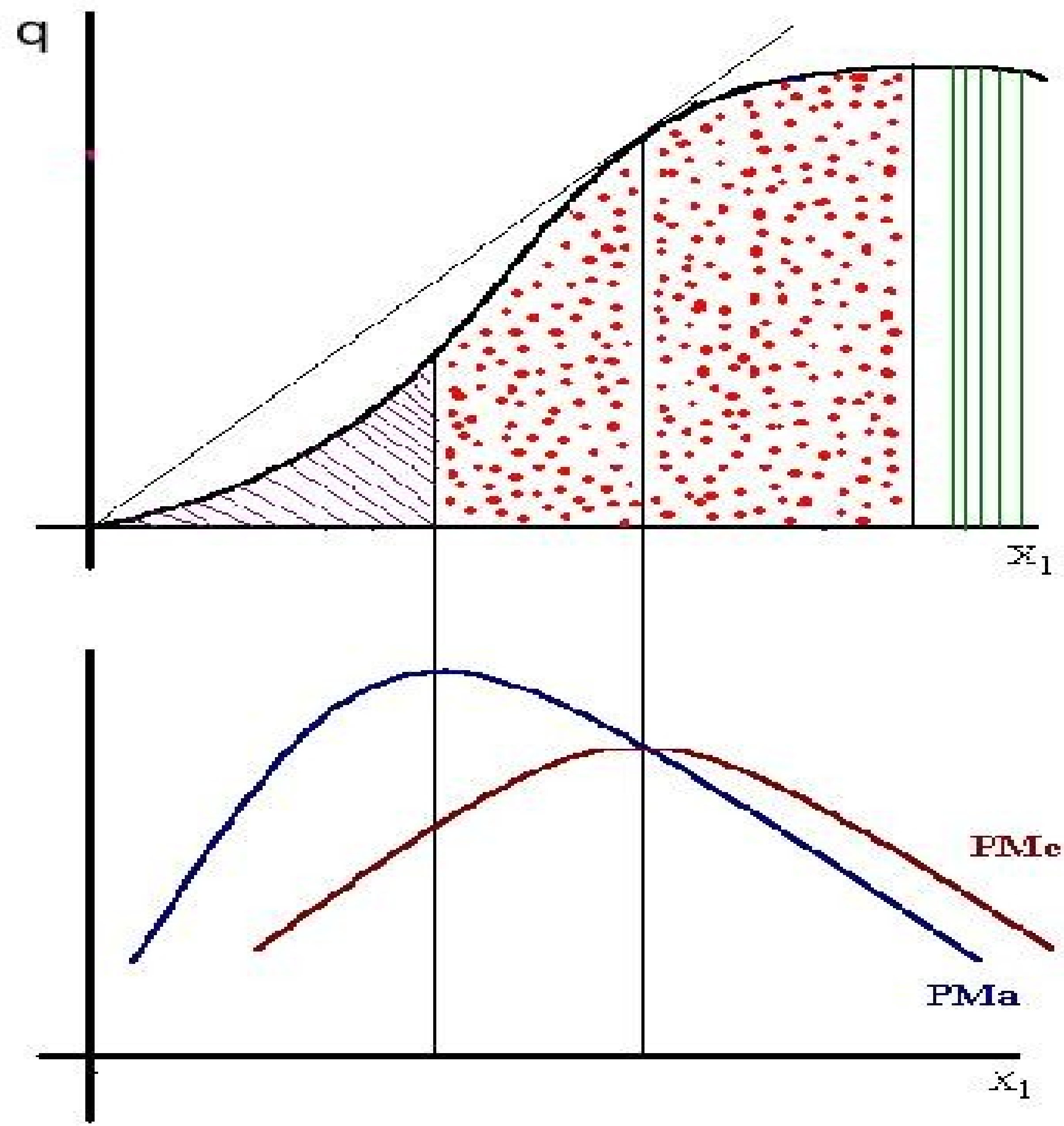
$$PMe_1 = \frac{f(X_{1,} X_2)}{X_1}$$

$$\frac{\delta PMe_1}{\delta X_1} = \frac{\delta f(X_{1,} X_2)}{\delta X_1} = 0$$

$$\frac{\delta PMe_1}{\delta X_1} = \frac{X_1 PMg_1 - f(X_1, X_2)}{X_1^2} = 0$$

$$X_1 PMg_1 = f(X_1, X_2) \rightarrow PMg_1 = \frac{f(X_1, X_2)}{X_1} = PMe_1$$

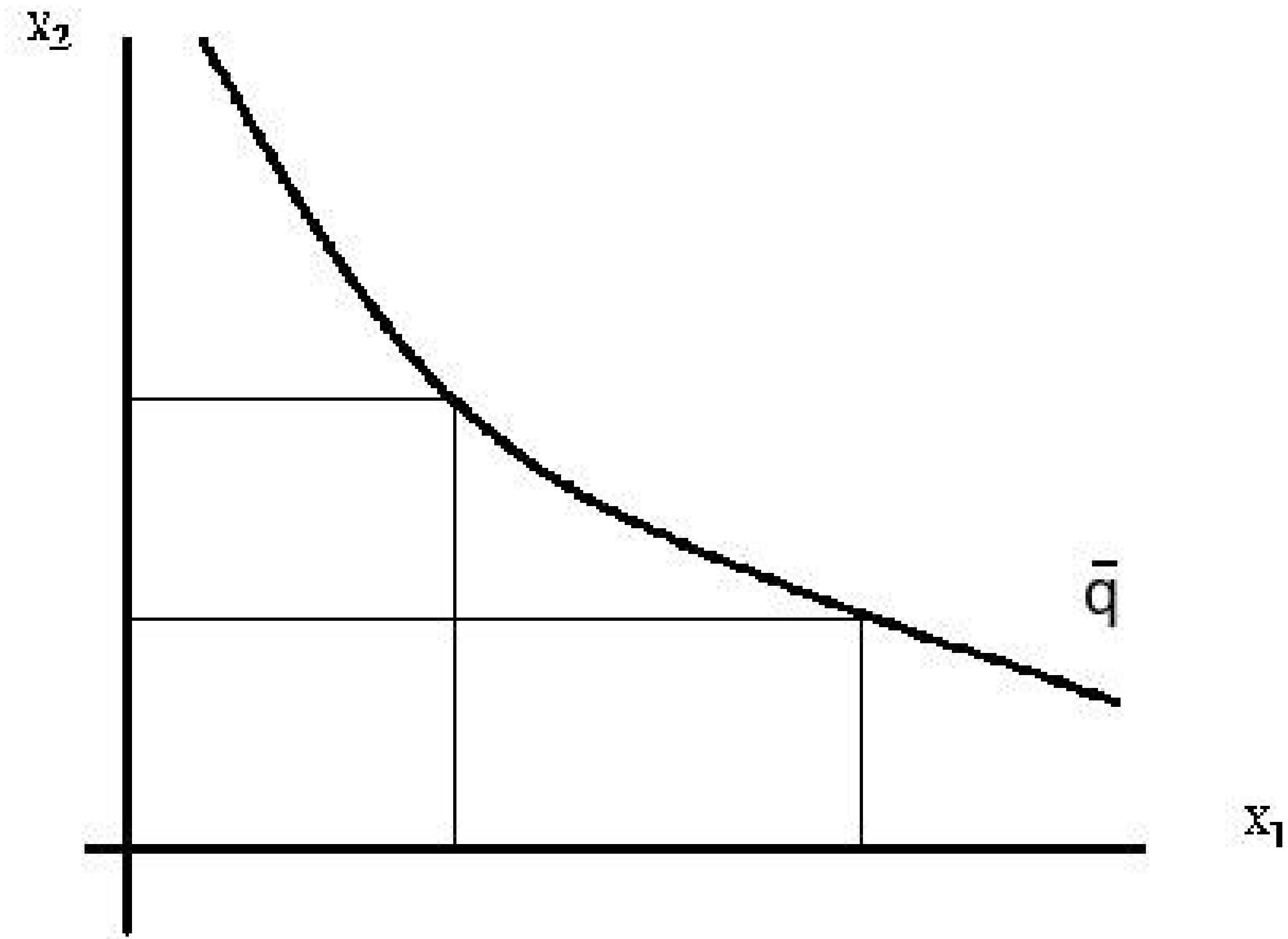

q
x_1



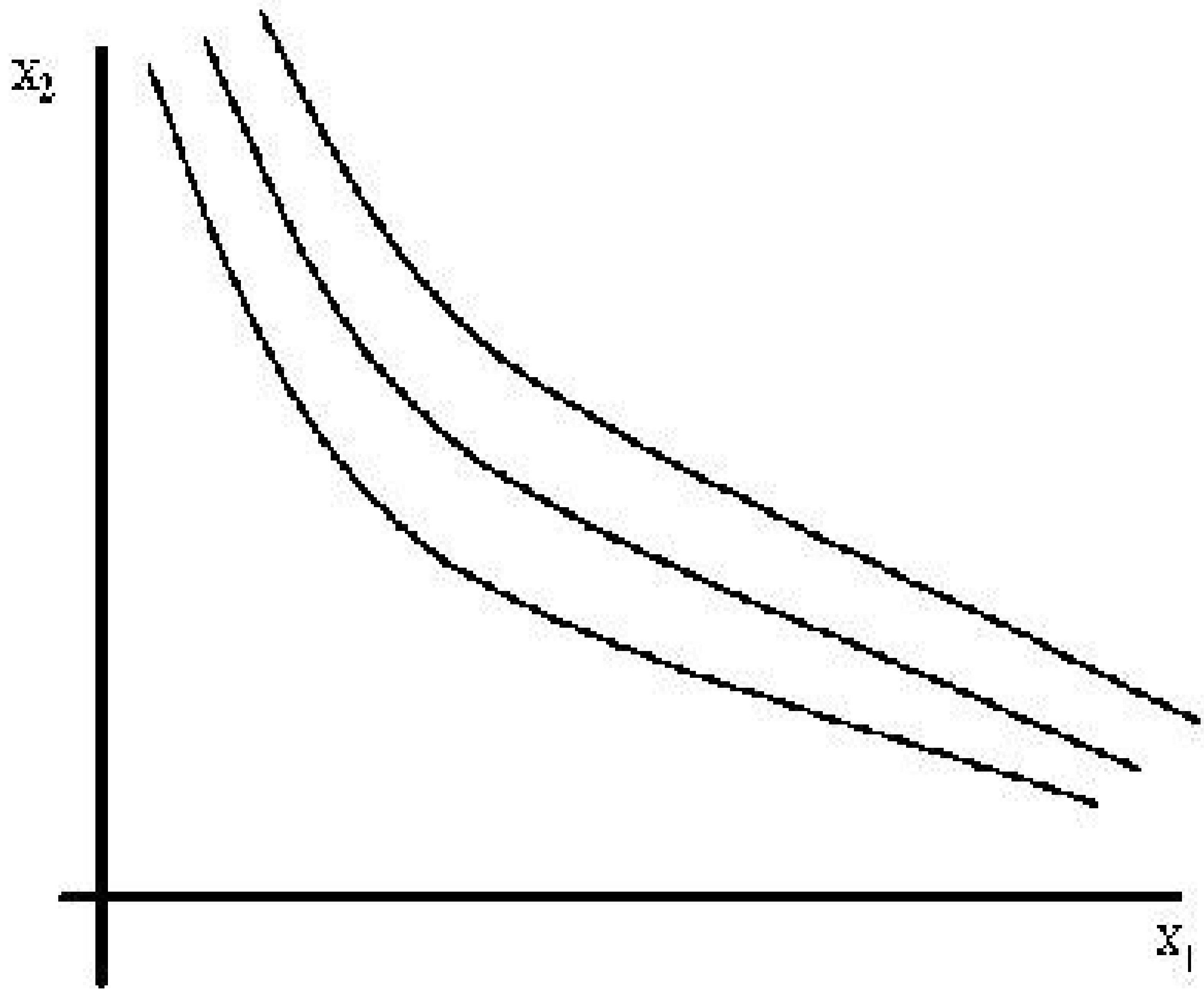
q
x1
PMc
PMa
x1

ISOCUANTAS

$$\bar{q} = f\left(X_{1,}X_{2}\right)$$

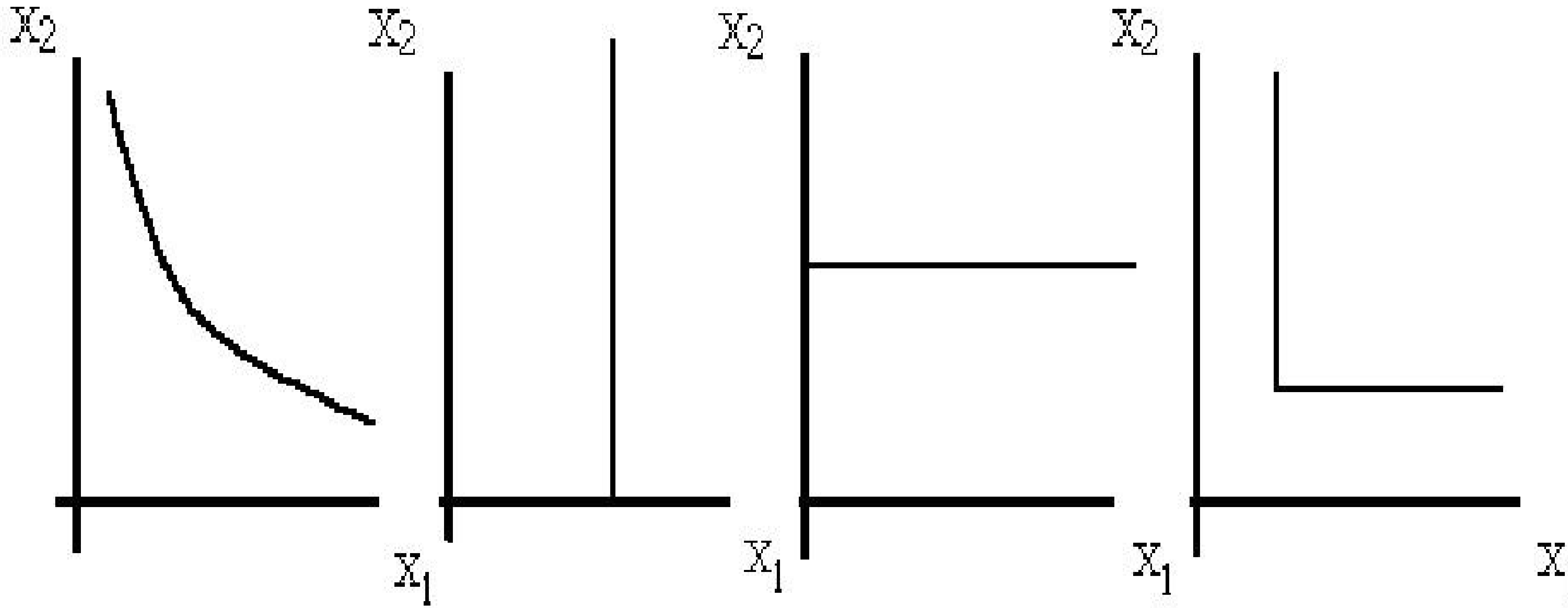


x_2
$\bar{q}$
x_1



X2
X1

TIPOLOGÍA


X_2
X_1
X_2
X_1
X_2
X_1
X_2
X_1

SUSTITUIBILIDAD DE FACTORES

$$\bar{q} = f\left(X_{1,} X_2\right)$$

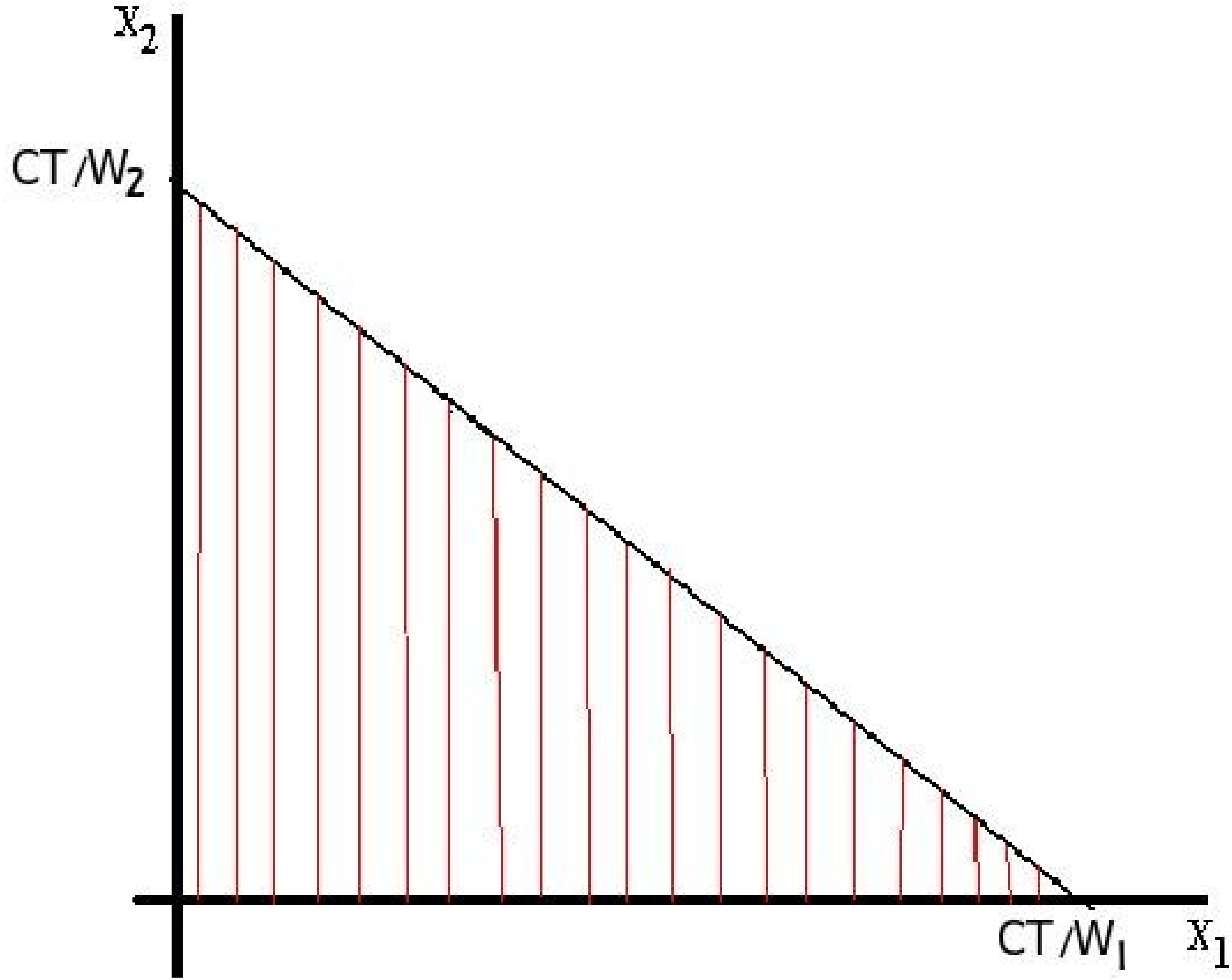
$$\delta \bar{q} = \frac{\delta f\left(X_{1,} X_2\right)}{\delta X :_1} dX_1 + \frac{\delta f\left(X_{1,} X_2\right)}{\delta X :_2} dX_2 = 0$$

$$PMg_1 \; dX_1 + PMg_2 \; dX_2 = 0$$

$$-\frac{dX_2}{dX_1} = \frac{PMg_1}{PMg_2} = TTSF$$

La función de costos

$$CT = W_1 X_1 + W_2 X_2$$



X2
CT /W2
CT /W1
X1

Costo Medio

$$CMe = \frac{CT}{q}$$

Costo Marginal

$$CMg = \frac{dCT}{dq}$$

¿Cómo se relacionan el CMe y el CMg?

Vamos a buscar el nivel de producción que minimiza el costo medio

$$CMe = \frac{CT}{q}$$

$$\frac{dCMe}{dq}=0$$

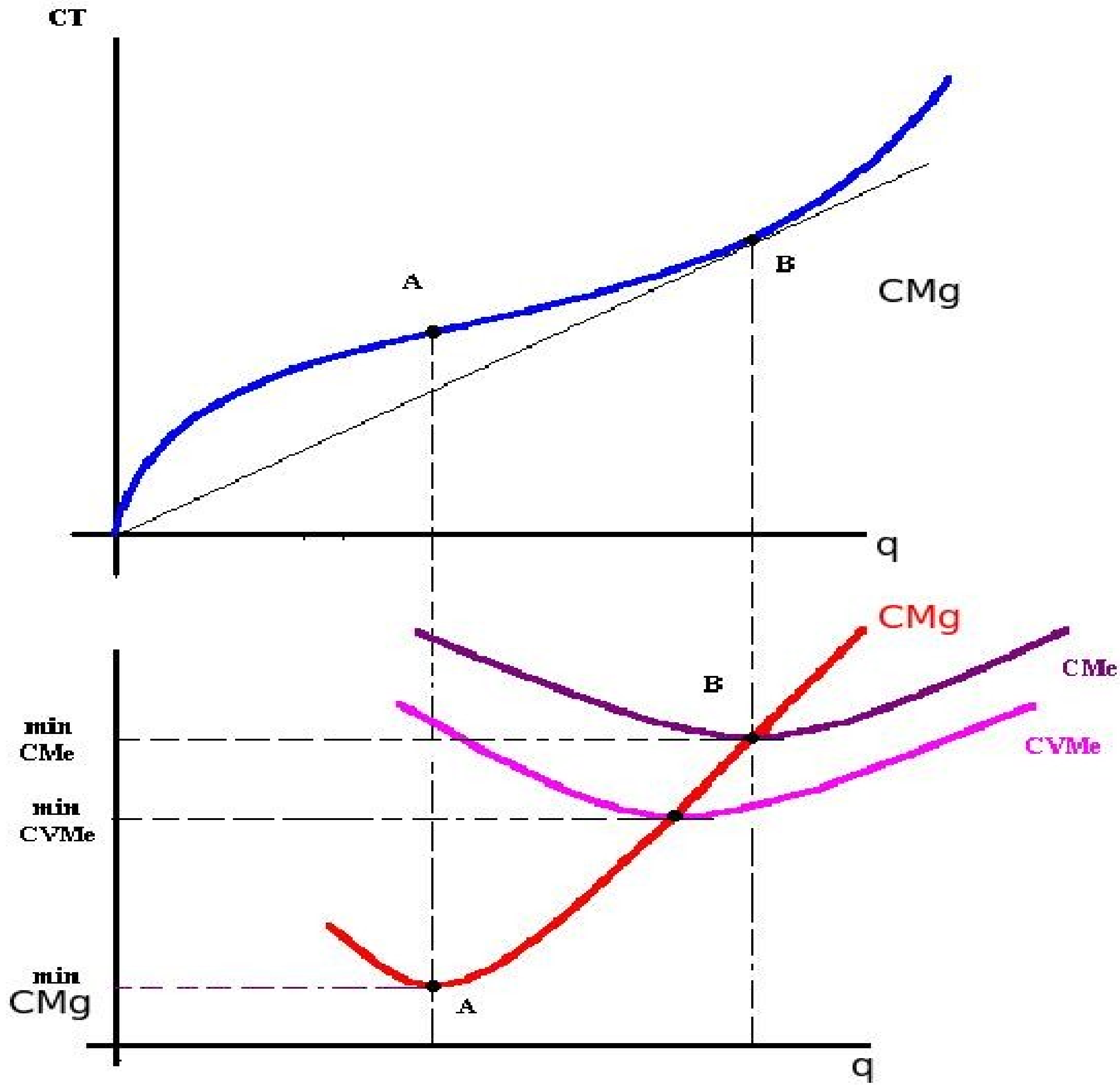
$$\frac{dCMe}{dq}=\frac{d\left(\frac{CT}{q}\right)}{dq}=\frac{q\,CMg-CT}{q^{2}}=0$$

$$qCMg=CT\rightarrow CMg=\frac{CT}{q}=CMe$$

Pero también

$$CMg = \frac{CV}{q} = CVMe$$

La curva de costo marginal corta las curvas de costo variable medio y costo medio cuando éstas se encuentran en su valor mínimo


CT
A
B
CMg
q
CMg
CMe
CVMe
B
min
CMe
min
CVMe
min
CMg
A
q

Primal

Maximizar q sujeto a la restricción de costos

$$max\, q = f\left(X_{1,} X_{2}\right)\ s.a.\ CT = W_1 X_1 + W_2 X_2$$

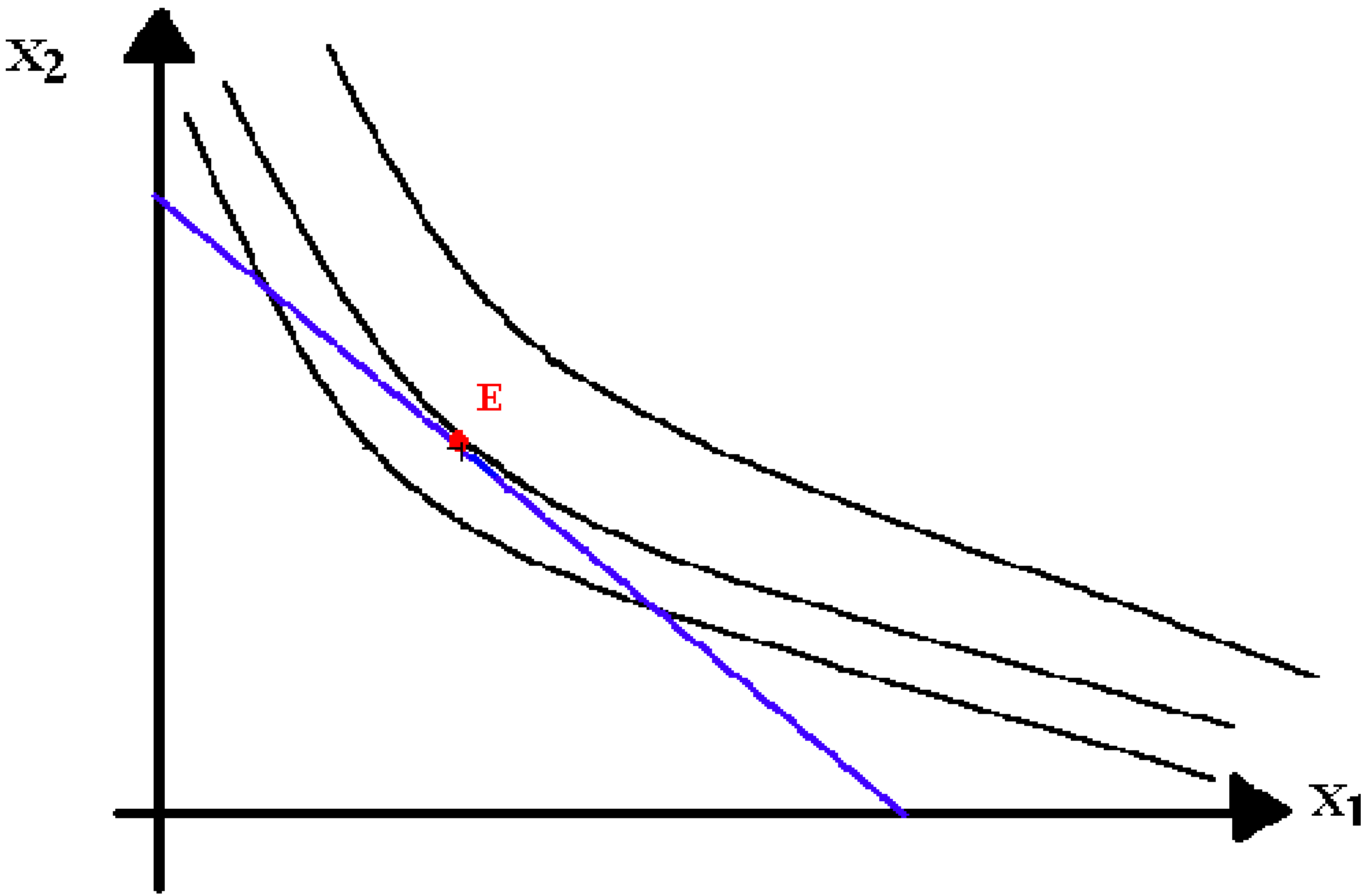
$$max\, L = f\left(X_{1,} X_{2}\right) + \lambda\left(CT - W_1 X_1 + W_2 X_2\right)$$

$$\frac{\delta L}{\delta X_1} = PMg_1 - \lambda W_1 = 0 \rightarrow \lambda = \frac{PMg_1}{W_1}$$

$$\frac{\delta L}{\delta X_2} = PMg_2 - \lambda W_2 = 0 \rightarrow \lambda = \frac{PMg_2}{W_2}$$

$$\frac{\delta L}{\delta \lambda} = CT - W_1 X_1 - W_2 X_2 = 0 \rightarrow CT = W_1 X_1 + W_2 X_2$$

$$\frac{PMg_1}{PMg_2} = \frac{W_1}{W_2}$$



Dual

Minimizar CT sujeto a la restricción de la producción

$$min\,CT = W_1 X_1 + W_2 X_2 \ \ s.a.\ \ \bar{q} = f\left(X_{1,} X_2\right)$$

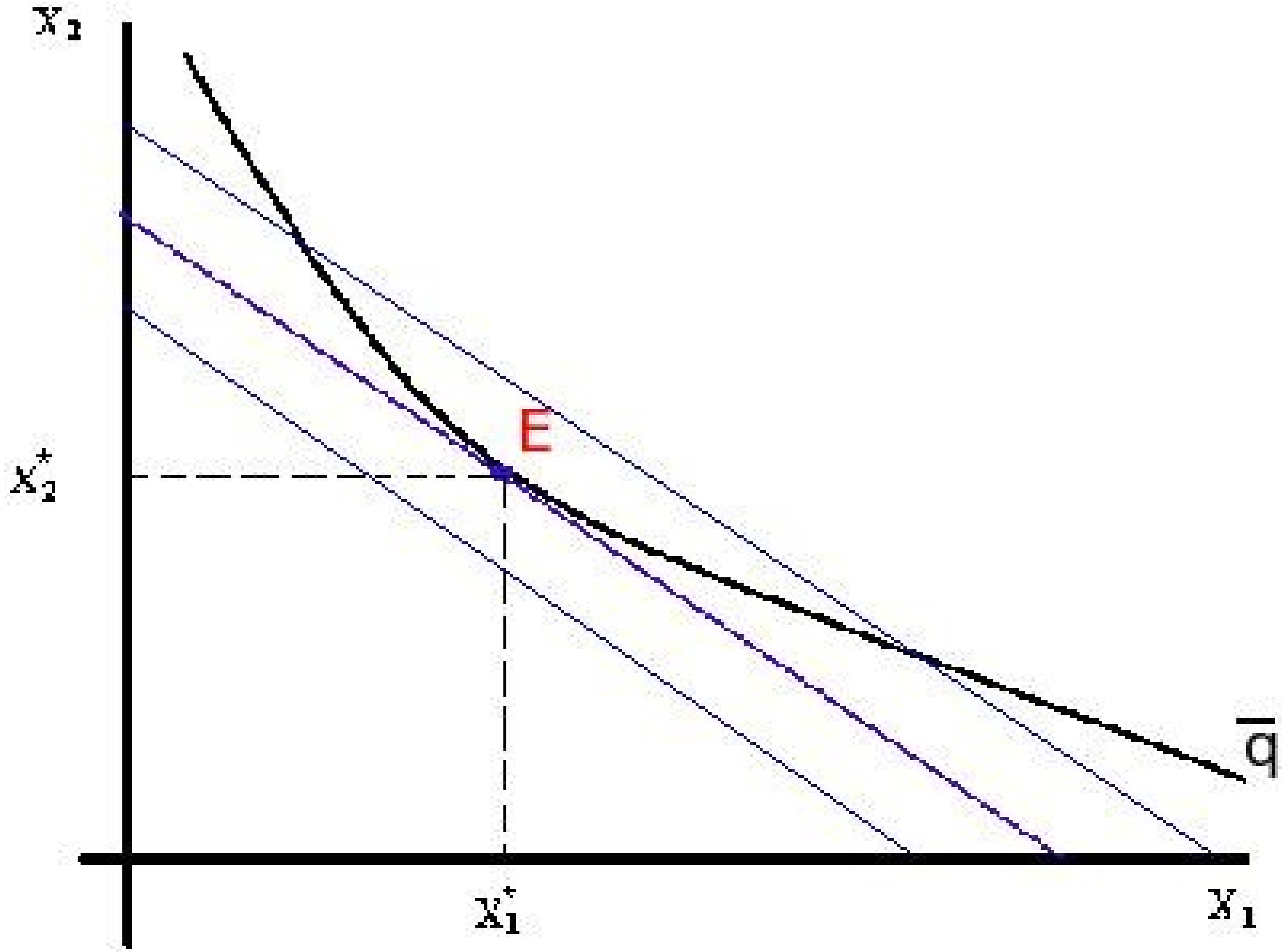
$$min\,L = W_1 X_1 + W_2 X_2 + \mu\left(\bar{q} - f\left(X_{1,} X_2\right)\right)$$

$$\frac{\delta L}{\delta X_1} = W_1 - \mu\,PMg_1 = 0 \rightarrow \mu = \frac{W_1}{PMg_1}$$

$$\frac{\delta L}{\delta X_2} = W_2 - \mu\,PMg_2 = 0 \rightarrow \mu = \frac{W_2}{PMg_2}$$

$$\frac{\delta L}{\delta \mu} = \bar{q} - f\left(X_1, X_2\right)$$

$$\frac{W_1}{W_2} = \frac{PMg_1}{PMg_2}$$



Interpretación económica de μ

$$\mu = \frac{W_1}{PMg_1} \rightarrow PMg_1 = \frac{W_1}{\mu}$$

$$\mu = \frac{W_2}{PMg_2} \rightarrow PMg_2 = \frac{W_2}{\mu}$$

$$\delta\, CT = W_1\, dX_1 + W_2\, dX_2$$

$$\delta\, q = PMg_1\, dX_1 + PMg_2\, dX_2$$

$$\delta\, q = \frac{W_1}{\mu}\, dX_1 + \frac{W_2}{\mu}\, dX_2$$

$$\delta q = \frac{1}{\mu}[W_1 dX_1 + W_2 dX_2] = \frac{1}{\mu}[\delta CT]$$

$$\rightarrow \mu = \frac{\delta CT}{\delta q}$$

$$\rightarrow \mu = CMg$$

La función de costos

$$CT = f\left(W_{1,} W_{2,} q\right) = W_1 X_1^{CT}\left(W_1, W_{2,} q\right) + W_2 X_2^{CT}\left(W_1, W_{2,} q\right)$$

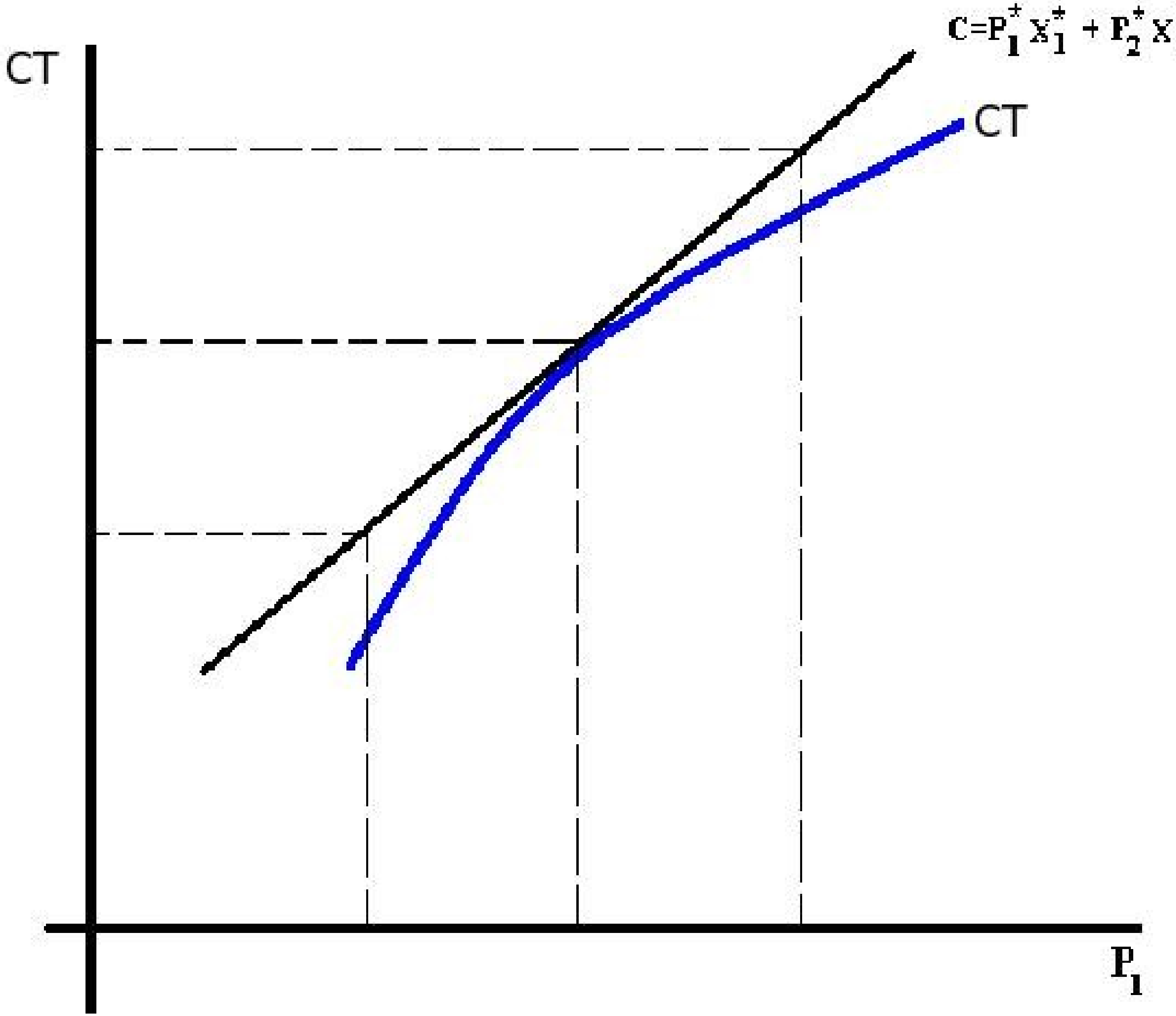
propiedades

$$X_i^{CT} = X_i^{CT}(W_1, W_2, q) \Longrightarrow$$ **Homogenea de grado cero en precios de los factores**

$$CT(W_1, W_2, q) \Longrightarrow$$ **Homogenea de grado uno**

$$CT(W_1, W_2, q) \Longrightarrow$$ **Cóncava en precios de los factores**

$$\frac{\delta CT}{\delta P_i} = X_i^{CT}(W_1, W_2, q) \Longrightarrow$$ **Lema de Shepard**


CT
$C = P_1^* X_1^* + P_2^* X_2^*$
CT
P_1